
GCSE Mathematics

Algebraic Proof

Mark Scheme - Higher | Grades 6-9

M = method mark, A = accuracy mark (dependent on the preceding M mark), B = independent mark.

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Edexcel-style questions

Q1 Grade 6 Edexcel style

Total: 3

Part	Mark scheme	Marks
(a)	let the two consecutive integers be n and $n + 1$	M1
(a)	$n + (n + 1) = 2n + 1$	M1
(a)	$2n + 1$ is one more than a multiple of 2, so the sum is always odd	A1

Q2 Grade 7 Edexcel style

Total: 3

Part	Mark scheme	Marks
(a)	let the three consecutive integers be n , $n + 1$, $n + 2$	M1
(a)	$n + (n + 1) + (n + 2) = 3n + 3$	M1
(a)	$= 3(n + 1)$, which is a multiple of 3 for any integer n	A1

Q3 Grade 8 Edexcel style

Total: 3

Part	Mark scheme	Marks
(a)	$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2$	M1
(a)	$= 2n + 1$	M1
(a)	so $(n + 1)^2 - n^2 = 2n + 1$, as required	A1

Q4 Grade 8 Edexcel style

Total: 3

Part	Mark scheme	Marks
(a)	the reversed number is $10b + a$	M1
(a)	$(10a + b) + (10b + a) = 11a + 11b$	M1
(a)	$= 11(a + b)$, which is a multiple of 11 for any digits a and b	A1

Q5 Grade 9 Edexcel style

Total: 3

Part	Mark scheme	Marks
(a)	choose $n = 4$	B1
(a)	$4^2 + 4 + 1 = 21$	M1
(a)	$21 = 3 \times 7$, which is not prime, so the statement is false	A1

AQA-style questions

Q6 Grade 6 AQA style

Total: 3

Part	Mark scheme	Marks
(a)	let the three consecutive even numbers be $2n$, $2n + 2$, $2n + 4$	M1
(a)	$2n + (2n + 2) + (2n + 4) = 6n + 6$	M1
(a)	$6n + 6 = 6(n + 1)$, which is a multiple of 6 for any integer n	A1

Q7 Grade 7 AQA style

Total: 3

Part	Mark scheme	Marks
(a)	$n^2 + n = n(n + 1)$	M1
(a)	n and $n + 1$ are consecutive integers, so exactly one of them is always even	M1
(a)	an even number multiplied by any integer is even, so $n^2 + n$ is always even	A1

Q8 Grade 8 AQA style

Total: 4

Part	Mark scheme	Marks
(a)	let the consecutive odd numbers be $2n + 1$ and $2n + 3$	M1
(a)	$(2n + 3)^2 - (2n + 1)^2 = (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$	M1
(a)	$= 8n + 8$	M1
(a)	$= 8(n + 1)$, which is a multiple of 8 for any integer n	A1

Q9 Grade 8 AQA style

Total: 3

Part	Mark scheme	Marks
(a)	the reversed number is $10b + a$	M1
(a)	$(10a + b) - (10b + a) = 9a - 9b$	M1
(a)	$= 9(a - b)$, which is a multiple of 9 for any digits a and b	A1

Q10 Grade 9 AQA style

Total: 3

Part	Mark scheme	Marks
(a)	$n^2 + (n + 1)^2 = n^2 + n^2 + 2n + 1$	M1
(a)	$= 2n^2 + 2n + 1$	M1
(a)	$= 2(n^2 + n) + 1$, which is odd for any integer n	A1

OCR-style questions

Q11 Grade 6 OCR style

Total: 3

Part	Mark scheme	Marks
(a)	let the two odd numbers be $2n + 1$ and $2m + 1$	M1
(a)	$(2n + 1)(2m + 1) = 4nm + 2n + 2m + 1$	M1
(a)	$= 2(2nm + n + m) + 1$, which is odd for any integers n and m	A1

Q12 Grade 7 OCR style

Total: 2

Part	Mark scheme	Marks
(a)	$6n + 9 = 3(2n + 3)$	M1
(a)	$3(2n + 3)$ is a multiple of 3 for any integer n	A1

Q13 Grade 8 OCR style

Total: 3

Part	Mark scheme	Marks
(a)	$(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$	M1
(a)	$= 8n$	M1
(a)	so $(2n + 1)^2 - (2n - 1)^2 = 8n$, as required	A1

Q14 Grade 8 OCR style

Total: 3

Part	Mark scheme	Marks
(a)	$100d + 10d + d = 111d$	M1
(a)	$111 = 3 \times 37$	M1
(a)	so $111d = 3 \times (37d)$, which is a multiple of 3 for any digit d	A1

Q15 Grade 9 OCR style

Total: 4

Part	Mark scheme	Marks
(a)	let the consecutive multiples of 3 be $3n$ and $3n + 3$	M1
(a)	$(3n + 3)^2 - (3n)^2 = (9n^2 + 18n + 9) - 9n^2$	M1
(a)	$= 18n + 9$	M1
(a)	$= 9(2n + 1)$, which is a multiple of 9 for any integer n	A1

Teachably-style questions

Q16 Grade 6 Teachably style

Total: 3

Part	Mark scheme	Marks
(a)	let the odd number be $2n + 1$ and the even number be $2m$	M1
(a)	$(2n + 1) + 2m = 2n + 2m + 1$	M1
(a)	$= 2(n + m) + 1$, which is odd for any integers n and m	A1

Q17 Grade 7 Teachably style

Total: 3

Part	Mark scheme	Marks
(a)	let the two multiples of 5 be $5a$ and $5b$	M1
(a)	$5a + 5b = 5(a + b)$	M1
(a)	which is a multiple of 5 for any integers a and b	A1

Q18 Grade 8 Teachably style

Total: 3

Part	Mark scheme	Marks
(a)	$n^2 - (n - 2)^2 = n^2 - (n^2 - 4n + 4)$	M1
(a)	$= 4n - 4$	M1
(a)	so $n^2 - (n - 2)^2 = 4n - 4$, as required	A1

Q19 Grade 8 Teachably style

Total: 3

Part	Mark scheme	Marks
(a)	$1000a + 100b + 10a + b = 1010a + 101b$	M1
(a)	$= 101(10a + b)$	M1
(a)	which is a multiple of 101 for any digits a and b	A1

Q20 Grade 9 Teachably style

Total: 4

Part	Mark scheme	Marks
(a)	$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$	M1
(a)	$n(n - 1)(n + 1)$ is the product of three consecutive integers	M1
(a)	among any three consecutive integers, at least one is a multiple of 2 and exactly one is a multiple of 3	M1
(a)	so the product is always a multiple of $2 \times 3 = 6$	A1

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